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THREE DIMENSIONAL STRESS STATE OF SMART COMPOSITE LAMINATED CROSS PLY USING HSDT WITH ZIG ZAG FUNCTION

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ABSTRACT
Thermal analysis of smart composite laminated plate using higher order theory with zig zag function. To develop the analytical procedure and to investigate the thermal characteristics for a three dimensional cross ply laminated plate. The material is considered to be orthotropic under thermal load based on higher order displacement model with zig-zag function, without enforcing zero transverse shear stress on the top and bottom faces of the laminated plates. The related functions and derivations for equation of motion are obtained using the dynamic version of the principle of virtual work or Hamilton's principle. The solutions are obtained by using Navier's and numerical methods for anti-symmetric cross-ply with specific type of simply supported boundary conditions SS. Computer programs have been developed to find the stresses and deflections for various aspect ratios, side thickness ratios (a/h) and voltage.

KEYWORDS: HSDT-higher order shear deformation theory; Hamilton's principle; Navier's Strokes equation; C++; SS- simply supported.

INTRODUCTION
In this work actuator is coupled to the top of the laminated composite material plates to achieve its thermal characteristics, which are tabulated in non-dimensional form of various aspect ratios, side thickness ratios (a/h) and voltages, and also to evaluate electric potential function by solving higher order differential equation satisfying electric boundary conditions along the thickness direction of piezoelectric layer. These solutions are plotted as a function of aspect ratio, thickness ratio (z/h) and voltages etc. The effects of shear deformation, coupling, degree of orthotropy and voltage on the response of smart composite laminated plates are investigated.

FORMULATION AND DEVELOPMENT OF DISPLACEMENT MODEL
A laminated plate of $0 \le x \le a$; $0 \le y \le b$ and $-h/2 \le z \le h/2$ is considered. The displacement components u (x, y, z, t), v (x, y, z, t) and w (x, y, z, t) at any point in the plate space are expanded in terms of thickness coordinate.The elasticity solution indicates that the transverse shear stress vary parabolically through the plate thickness. This requires the use of displacement field in which the in-plane displacements are expanded as cubic functions of the thickness coordinate in addition the transverse normal strain may vary in non-linearly trough the plate thickness. The higher-order displacement field with Zig-Zag function which satisfies the above criteria is assumed as:

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) + \theta_k s_1(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) + \theta_k s_2(x,y) \\ w(x,y,z) &= w_o(x,y) + z\theta_z(x,y) + z^2 w_o^*(x,y) + z^3\theta_z^*(x,y) \end{aligned}\right\} \quad (1)$$

Where
u0, v0 , s1, s2 are the in plane displacements of a point (x, y) at the mid plane.
wo is the transverse displacement of a point (x, y) at the mid plane.
θx, θy , θz are rotations of the normal to the mid plane about y and x–axes.

u0*, v0*, w0*, θx*, θy*, and θz* are the corresponding higher–order deformation terms

θ_k is the Zig-Zag function, defined as:

$$\theta_k = 2(-1)^k \frac{Z_k}{h_k}$$

Z_k is the local transverse coordinate with its origin at the centre of the kth layer and hk is the corresponding layer thickness.The Zig-Zag function is piecewise linear with values of –1 and 1 alternately at the different interfaces. The Zig-Zag function takes care of the slope discontinuities at the interfaces of the laminate. The use of a Zig-Zag function is more effective than a discrete layer approach of approximating the displacement variations over the thickness of each layer separately.

The strain components are:

$$\varepsilon_x = \varepsilon_{x0} + zk_{sx} + z^2\varepsilon^*_{x0} + z^3k^*_x$$

$$\varepsilon_y = \varepsilon_{y0} + zk_{sy} + z^2\varepsilon^*_{y0} + z^3k^*_y$$

$$\varepsilon_z = \varepsilon_{z0} + zk_z + z^2\varepsilon^*_{z0}$$

$$\gamma_{xy} = \varepsilon_{xyo} + zk_{sxy} + z^2\varepsilon^*_{xyo} + z^3k^*_{xy}$$

$$\gamma_{yz} = \phi_{sy} + zK_{yz} + z^2\phi^*_y + z^3k^*_{yz}$$

$$\gamma_{xz} = \phi_{sx} + zK_{xz} + z^2\phi^*_x + z^3k^*_{xz} \quad (2)$$

LAMINA CONSTITUTIVE RELATIONS

Each lamina in the laminate is assumed to be in a three dimensional stress state so that the constitutive relations for a typical lamina L with reference to the fiber–matrix coordinate axes (1-2-3) is written as:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{12}\\ \tau_{23}\\ \tau_{13}\end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix} \begin{Bmatrix}\varepsilon_1 - \alpha_1\Delta T\\ \varepsilon_2 - \alpha_2\Delta T\\ \varepsilon_3 - \alpha_3\Delta T\\ \gamma_{12}\\ \gamma_{23}\\ \gamma_{13}\end{Bmatrix} \quad (3)$$

Where

σ1, σ2, σ3, τ12, τ23, τ13 are the stresses

ε1, ε2, ε3,γ1γ23, γ13 are the linear strain components.

Cij’s are the plane stress reduced elastic constants of the Lth lamina.

$\alpha_1, \alpha_2, \alpha_3$ are the coefficients of thermal expansions.

ΔT= T (x, y, z, t) is the temperature increment from the reference.

The following relations hold between plane stress reduced elastic constants and engineering elastic constants.

$$C_{11} = \frac{E_1(1-\mu_{23}\mu_{32})}{\Delta} \quad C_{12} = \frac{E_1(\mu_{21}+\mu_{31}\mu_{23})}{\Delta} \quad C_{13} = \frac{E_1(\mu_{31}+\mu_{21}\mu_{32})}{\Delta}$$

$$C_{22} = \frac{E_2\left(1-\mu_{13}\mu_{31}\right)}{\Delta}\; C_{23} = \frac{E_2\left(\mu_{32}+\mu_{12}\mu_{31}\right)}{\Delta}\; C_{33} = \frac{E_3\left(1-\mu_{12}\mu_{21}\right)}{\Delta} \tag{4}$$

Where

$$\Delta = (1-\mu_{12}\mu_{21}-\mu_{23}\mu_{32}-\mu_{31}\mu_{13}-2\mu_{12}\mu_{23}\mu_{31})$$

LAMINATE CONSTITUTIVE EQUATIONS

In the laminate coordinates (x, y, z), the stress strain relations for the Lth lamina is written as:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\end{Bmatrix}^L = \begin{bmatrix}Q_{11} & Q_{12} & Q_{13} & 0\\ Q_{12} & Q_{22} & Q_{23} & 0\\ Q_{31} & Q_{32} & Q_{33} & 0\\ 0 & 0 & 0 & Q_{44}\end{bmatrix}^L \begin{Bmatrix}\in_x -\alpha_x\Delta T\\ \in_y -\alpha_y\Delta T\\ \in_z -\alpha_z\Delta T\\ \gamma_{xy} -\alpha_{xy}\Delta T\end{Bmatrix}^L$$

$$\begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix}^L = \begin{bmatrix}Q_{55} & Q_{56}\\ Q_{65} & Q_{66}\end{bmatrix}^L \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}^L \tag{5}$$

Where

σx, σy, σzτxy, τyz, τxz are the stresses and

ε x, ε y, ε z,γxy, γyz, γxzare the strains with respect to the laminate axes.

Qij's are the transformed elastic constants:

$$Q_{11} = C_{11}C^4 + 2(C_{12}+2C_{44})S^2C^2 + C_{22}S^4$$
$$Q_{22} = (C_{11}S^4 + C_{22}C^4 + (2C_{12}+4C_{44})S^2C^2$$
$$Q_{23} = (C_{13}S^2 + C_{23}C^2)$$
$$Q_{24} = (C_{11}-C_{12}-2C_{44})S^3C + (C_{12}-C_{22}+2C_{44})C^3S$$
$$Q_{33} = C_{33}$$
$$Q_{12} = C_{12}(C^4+S^4) + (C_{11}+C_{22}-4C_{44})S^2C^2$$
$$Q_{13} = (C_{11}-C_{12}-2C_{44})S\;C^3 + (C_{12}-C_{22}+2C_{44})CS^3$$
$$Q_{34} = (C_{31}-C_{32})SC$$
$$Q_{44} = (C_{11}-2C_{12}+C_{22}-2C_{44})S^2C^2 + C_{44}(C^4+S^4)$$
$$Q_{55} = C_{55}C^2 + C_{66}S^2$$
$$Q_{56} = (C_{66}-C_{55})CS$$
$$Q_{66} = (C_{55}S^2 + C_{66}C^2)$$
$$\alpha_x = \alpha_1 C^2 + \alpha_2 S^2$$
$$\alpha_y = \alpha_1 S^2 + \alpha_2 C^2$$
$$\alpha_Z = \alpha_1 S^2 + \alpha_2 C^2$$
$$\alpha_{xy} = (\alpha_1 - \alpha_2)SC \tag{6}$$

And Qij = Qji, i, j = 1 to 6

Where C = cosα, S = sin α

The stress strain relationship in global X – Y – Z coordinate is written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0\\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0\\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & Q_{55} & Q_{66}\\ 0 & 0 & 0 & 0 & Q_{65} & Q_{66}\end{bmatrix} \begin{Bmatrix}\varepsilon_x - \alpha_x \Delta T\\ \varepsilon_y - \alpha_y \Delta T\\ \varepsilon_z - \alpha_z \Delta T\\ \gamma_{xy} - \alpha_{xy}\Delta T\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{7}$$

The governing equations of displacement model are derived using the principle of virtual work or Hamilton's principle.

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0 \tag{8}$$

$$\delta U = \int_A \left\{ \int_{-h/2}^{h/2} \left[\sigma_x \delta \in_x + \sigma_y \delta \in_y + \sigma_z \delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz}\right] dz \right\} dx\, dy$$

$$\delta V = -\int q\,\delta w_0\, dx\, dy$$

$$\begin{aligned}\delta K = \int_A \Big\{ \int_{-h/2}^{h/2} \rho_0 \Big[&\left(\dot u_0 + Z\dot\theta_x + Z^2 \dot u_0^* + Z^3 \dot\theta_x^* + \theta_k \dot s_1\right)\left(\delta\dot u_0 + Z\delta\dot\theta_x + Z^2\, \delta\dot u_0^* + Z^3\, \delta\dot\theta_x^* + \theta_k \delta\dot s_1\right) + \\ &\left(\dot v_0 + Z\dot\theta_y + Z^2 \dot v_0^* + Z^3 \dot\theta_y^* + \theta_k \dot s_2\right)\left(\delta\dot v_0 + Z\delta\dot\theta_y + Z^2\delta\, \dot v_0^* + Z^3\delta\, \dot\theta_y^* + \theta_k \delta\dot s_2\right) + \\ &\left(\dot w_0 + Z\dot\theta_Z + Z^2\, \dot w_0^* + Z^3\, \dot\theta_Z^*\right)\left(\delta\dot w_0 + \delta Z\dot\theta_Z + Z^2\delta\, \dot w_0^* + Z^3\, \delta\dot\theta_Z^*\right)\, dz \Big\}\, dx\, dy\end{aligned}$$

Where

δU = Virtual strain energy

δV = Virtual work done by applied forces

δK = Virtual kinetic energy

δU + δV = Total potential energy.

q = distributed load over the surface of the laminate.

ρ0 = density of plate material.

$\dot u_0 = \partial u0/\partial t$, $\dot v_0 = \partial v0/\partial t$ etc. indicates the time derivatives.

The virtual work statement shown in Eq (8), integrating through the thickness of laminate, the in-plane and transverse force and moment resultant relations in the form of matrix obtained as:

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$$\begin{Bmatrix} N \\ N^* \\ -- \\ M \\ M^* \\ -- \\ Q \\ Q^* \end{Bmatrix} = \begin{bmatrix} A & | & B & | & 0 \\ B^t & | & D_b & | & 0 \\ 0 & | & 0 & | & D_s \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \varepsilon_0^* \\ -- \\ K_s \\ K^* \\ -- \\ \phi_s \\ \phi^* \end{Bmatrix} - \begin{Bmatrix} N^T \\ N^{*T} \\ -- \\ M^T \\ M^{*T} \\ -- \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} N^{PZ} \\ N^{*PZ} \\ -- \\ M^{PZ} \\ M^{*PZ} \\ -- \\ Q^{PZ} \\ Q^{*PZ} \end{Bmatrix} \tag{9}$$

Where

$N = [N_x \ N_y \ N_Z \ N_{xy}]^t$; $N^* = [N_x^* \ N_y^* \ N_Z^* \ N_{xy}^*]^t$

N, N* are called the in-plane force resultants.

$M = [M_x \ M_y M_Z \ M_{xy}]^t$; $M^* = [M_x^* \ M_y^* \ M_Z \ M_{xy}^*]^t$

M, M* are called moment resultants.

$Q = [Q_x \ Q_y S_X S_Y]^t$; $Q^* = [S_x^* \ S_y^* \ Q_x^* \ Q_y^*]^t$

Q, Q* denotes the transverse force resultants.
The thermal force resultants:

$$\{N^T\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]^L \{\alpha\}^L \Delta T \ dz$$

$$\{N^{*T}\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]^L \{\alpha\}^L \Delta T z^2 \ dz$$

The thermal moments:

$$\{M^T\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]^L \{\alpha\}^L \Delta T \ z \ dz$$

$$\{M^{*T}\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]^L \{\alpha\}^L \Delta T \ z^3 \ dz$$

$N^{pz} = [N_x^{pz}, N_y^{pz}, N_z^{pz}, N_{xy}^{pz}]^t$; $N^{*pz} = [N_x^{*pz}, N_y^{*pz}, N_z^{pz}, N_{xy}^{*pz}]^t$

N^{pz}, N^{*pz} are in-plane piezo force resultants.

$M^{pz} = [M_x^{pz}, M_y^{pz}, M_z^{pz}, M_{xy}^{pz}]^t$; $M^{*pz} = [M_x^{*pz}, M_y^{*pz}, M_z^{pz}, M_{xy}^{*pz}]^t$

M^{pz}, M^{*pz} are piezo moment resultants.

$Q^{pz} = [Q_x^{pz} \ Q_y^{pz} S_x^{pz} S_y^{pz}]^t$; $Q^{*pz} = [S_x^{*pz} \ S_y^{*pz} \ Q_x^{*pz} \ Q_y^{*pz}]^t$

Q^{pz}, Q^{*pz} are transverse piezo force resultant.

$\varepsilon_0 = [\varepsilon_{x0} \ \varepsilon_{y0} \ \varepsilon_{z0} \ \varepsilon_{xy0}]^t$; $\varepsilon_0^* = [\varepsilon_{x0}^* \ \varepsilon_{y0}^* \ \varepsilon_{z0}^* \ \varepsilon_{xy0}^*]^t$

$k_s = [k_{sx} \ k_{sy} \ k_z \ k_{sxy}]^t$; $k^* = [k_x^* \ k_y^* \ k_z^* \ k_{xy}^*]^t$

$\phi_s = [\phi_{sx} \ \phi_{sy} \ k_{xz} \ k_{yz}]^t$; $\phi^* = [\phi_x^* \ \phi_y^* \ k_{xz}^* \ k_{yz}^*]^t$

For non-isothermal case, the thermal force resultants are

$$\{N^T\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]^L \{\alpha\}^L \Delta T \, dz$$

$$\{N^{*T}\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]^L \{\alpha\}^L \Delta T \, z^2 \, dz$$

$$\{M^T\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]^L \{\alpha\}^L \Delta T \, z \, dz$$

$$\{M^{*T}\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]^L \{\alpha\}^L \Delta T \, z^3 \, dz$$

Where

$$\{N^T\} = \{N_x^T N_y^T N_z^T N_{xy}^T\}^t \quad \{N^{*T}\} = \{N_x^{*T} N_y^{*T} N_z^{*T} N_{xy}^{*T}\}^t$$

$$\{M^T\} = \{M_x^T M_y^T M_z^T M_{xy}^T\}^t \quad \{M^{*T}\} = \{M_x^{*T} M_y^{*T} M_z^{*T} M_{xy}^{*T}\}^t$$

Using Hamilton's principle for total potential energy and by Equating the coefficients of each of virtual displacements δu0 ,δv0, δw0 ,δθx ,δθy ,δθz, δu0*, δv0*,δw0*,δθx*, δθy*, δθz*,δs1,δs2to zero, the following equations of motion are obtained:

$$\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1 \ddot{u}_0 + I_2\left(\ddot{\theta}_x + R\ddot{s}_1\right) + I_3 \ddot{u}_0^* + I_4 \ddot{\theta}_x^*$$

$$\delta v_0 : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_1 \ddot{v}_0 + I_2\left(\ddot{\theta}_y + R\ddot{s}_2\right) + I_3 \ddot{v}_0^* + I_4 \ddot{\theta}_y^*$$

$$\delta w_0 : \frac{\partial \theta_x}{\partial x} + \frac{\partial \theta_y}{\partial y} + P_z^+ = I_1 \ddot{w}_0 + I_2 \ddot{\theta}_z + I_3 \ddot{w}_0^* + I_4 \ddot{\theta}_z^*$$

$$\delta\theta_x : \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} + Q_x = I_2 \ddot{u}_0 + I_3\left(\ddot{\theta}_x + R\ddot{s}_1\right) + I_4 \ddot{u}_0^* + I_5 \ddot{\theta}_x^*$$

$$\delta\theta_y : \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} + Q_y = I_2 \ddot{v}_0 + I_3\left(\ddot{\theta}_y + R\ddot{s}_2\right) + I_4 \ddot{v}_0^* + I_5 \ddot{\theta}_y^*$$

$$\delta\theta_z : \frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y} + N_z + \frac{h}{2}(p_z^+) = I_2 \ddot{w}_0 + I_3 \ddot{\theta}_z + I_4 \ddot{w}_0^* + I_5 \ddot{\theta}_z^*$$

$$\delta u_0^* : \frac{\partial N_x^*}{\partial x} + \frac{\partial N_{xy}^*}{\partial y} + 2S_x = I_3 \ddot{u}_0 + I_4\left(\ddot{\theta}_x + R\ddot{s}_1\right) + I_5 \ddot{u}_0^* + I_6 \ddot{\theta}_x^*$$

$$\delta v_0^* : \frac{\partial N_y^*}{\partial y} + \frac{\partial N_{xy}^*}{\partial x} + 2S_y = I_3 \ddot{v}_0 + I_4\left(\ddot{\theta}_y + R\ddot{s}_2\right) + I_5 \ddot{v}_0^* + I_6 \ddot{\theta}_y^*$$

$$\delta w_0^* : \frac{\partial Q_x^*}{\partial x} + \frac{\partial Q_y^*}{\partial y} + 2M_z + \frac{h^2}{4}\left(p_z^+\right) = I_3 \ddot{w}_0 + I_4 \ddot{\theta}_z + I_5 \ddot{w}_0^* + I_6 \ddot{\theta}_z^*$$

$$\delta\theta_x^* : \frac{\partial M_x^*}{\partial x} + \frac{\partial M_{xy}^*}{\partial y} + 3Q_x^* = I_4 \ddot{u}_0 + I_5\left(\ddot{\theta}_x + R\ddot{s}_1\right) + I_6 \ddot{u}_0^* + I_7 \ddot{\theta}_x^*$$

$$\delta\theta_z^*: \frac{\partial S_x^*}{\partial x} + \frac{\partial S_y^*}{\partial y} + \frac{h^3}{8} q - 3N_z^* = I_4 \ddot{w}_0 + I_5 \ddot{\theta}_z + I_6 \ddot{w}_0^* + I_7 \ddot{\theta}_z^*$$

$$\delta\theta_y^*: \frac{\partial M_y^*}{\partial y} + \frac{\partial M_{xy}^*}{\partial x} + 3Q_y^* = I_4 \ddot{v}_0 + I_5 \left(\ddot{\theta}_y + R\ddot{s}_2\right) + I_6 \ddot{v}_0^* + I_7 \ddot{\theta}_y^*$$

$$\delta s_2: \frac{\partial RM_y}{\partial y} + \frac{\partial RM_{xy}}{\partial x} - RQ_y = RI_2 \ddot{v}_0 + RI_3 \left(\ddot{\theta}_y + R\ddot{s}_2\right) + RI_4 \ddot{v}_0^* + RI_5 \ddot{\theta}_y^*$$

$$\delta s_1: \frac{\partial RM_x}{\partial x} + \frac{\partial RM_{xy}}{\partial y} - RQ_x = RI_2 \ddot{u}_0 + RI_3 \left(\ddot{\theta}_x + R\ddot{s}_1\right) + RI_4 \ddot{u}_0^* + RI_5 \ddot{\theta}_x^*$$

(10)

The Navier's solutions of simply supported anti symmetric cross ply laminated plates.
The SS boundary conditions for the anti-symmetric cross ply laminated plates are:
At edges x = 0 and x = a
v0 =0, wo =0, θy =0, θz = 0, Mx =0, v0* =0, w0* =0, θy* =0, θz* = 0, Mx* = 0, Nx = 0, Nx* = 0,S2 =0 (11a)
At edges y = 0 and y = b
u0=0, wo =0, θx =0, θz =0, My =0, u0* =0, w0* =0,θx* =0, θz* =0, My* =0, Ny =0, Ny* = 0, S1 = 0 (11b)

The SS boundary conditions in Eq. (10) are considered for solutions of anti-symmetric cross-ply laminates using a higher order shear deformation theory with Zig-Zag function. The displacements at the mid plane will be defined to satisfy the boundary conditions in Eq. (10). These displacements will be substituted in governing equations to obtain the equations in terms of A, B, D parameters. The obtained equations will be solved to find the behaviour of the laminated composite plates. The boundary conditions in Eq. (10) are satisfied by the following expansions:

$$u_0(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}(t)\cos\alpha x \sin\beta y$$

$$v_0(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}(t)\sin\alpha x \cos\beta y$$

$$w_0(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}(t)\sin\alpha x \sin\beta y$$

$$\theta_x(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn}(t)\cos\alpha x \sin\beta y$$

$$\theta_y(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Y_{mn}(t)\sin\alpha x \cos\beta y$$

$$\theta_z(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Z_{mn}(t)\sin\alpha x \sin\beta y$$

$$u_0^*(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}^*(t)\cos\alpha x \sin\beta y$$

$$V_o^*(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}^*(t)\sin\alpha x \cos\beta y$$

$$w_0^*(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}^*(t)\sin\alpha x \sin\beta y$$

$$\theta_x^*(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn}^*(t)\ \cos\alpha x\ \sin\beta y$$

$$\theta_y^*(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Y_{mn}^*(t)\ \sin\alpha x\ \cos\beta y$$

$$\theta_z^*(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Z_{mn}^*(t)\ \sin\alpha x\ \sin\beta y$$

$$s_1(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} S_{1mn}(t)\ \cos\alpha x\ \sin\beta y$$

$$s_2(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} S_{2mn}(t)\ \sin\alpha x\ \cos\beta y \qquad (12)$$

RESULTS AND DISCUSSION

The material properties used for each orthotropic layer are

$E_1/E_2 = 2.5$ $G_{12}/E_2 = 0.5$ $G_{23}/E_2 = 0.2$ $E_2 = E_3 = 10^2\ N/m^2$

$\alpha_1 = 10^{-6}\ {}^0C^{-1}$ $\mu_{12} = \mu_{23} = \mu_{13} = 0.25$ $\alpha_2 = 1.125\times10^{-3}\ {}^0C^{-1}$ $\alpha_3 = 1.125\times10^{-3}\ {}^0C^{-1}$

$\alpha_{12} = 1.125\times10^{-3}\ {}^0C^{-1}$

PFRC Layer:

$C_{11} = 32.6Gpa$ $C_{12} = C_{21} = 4.3Gpa$ $C_{13} = C_{31} = 4.76Gpa$ $C_{22} = C_{23} = 7.2Gpa$

$C_{23} = 3.85Gpa$ $C_{44} = 1.05Gpa$ $C_{55} = C_{66} = 1.29Gpa$

$e_{31} = -6.76C/m^2$ $g_{11} = g_{22} = 0.037E-9C/Vm$ $g_{33} = 10.64E-9C/Vm$

The centre deflections and stresses are presented here in non-dimensional form using the following multipliers:

$m_1 = \dfrac{10e_2t^3}{t_0a_1^4}$, $m_2 = \dfrac{10}{t_0a_1^2}$, $m_3 = \dfrac{10}{t_0a_1^2}$, $m_4 = \dfrac{t^2}{t_0a_1^2}$, $m_5 = \dfrac{t}{t_0a_1}$

$m_6 = e_2\alpha_1t_0$, $m_7 = \dfrac{a_1^2t\alpha_1t_0}{t^2}$

The variation of non-dimensionalized transverse deflection (w) against side thickness ratios as a function of number of layers at voltage 100V for a simply supported cross ply smart composite laminated plates is showed in figure1.From figure the maximum transverse deflection is observed for 2 layers, and with increase in side to thickness ratio there is decrease in deflection. The deflection of the smart composite laminated plates decreases as the side of the piezoelectric actuators increases. The effect of coupling on deflections is quite significant for aspect ratio less than 3. Figure – 2 shows the variation of normal stresses as a function of side to thickness ratio (a/h) it shows normal stress is observed maximum for 2 layers. The effect of coupling is to decrease the stresses with the increase in aspect ratios.Figure-3 shows the maximum transverse shear stresses for simply supported cross-ply laminates as a function of side to thickness ratio. It is noted, transverse shear stress is maximum for 4layers. The effect of transverse shear stress deformation and coupling is quite significant for all values of side thickness ratio a/h < 5.

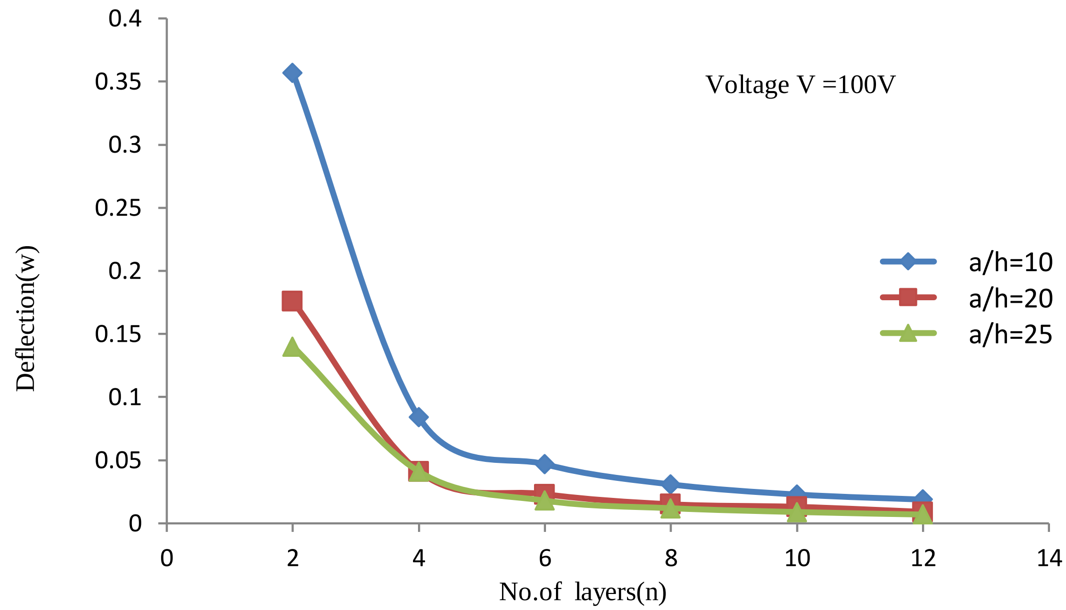

Fig.1.Non dimensionalised deflection vs no.of layers for a simply supported cross ply smart composite laminated plate.

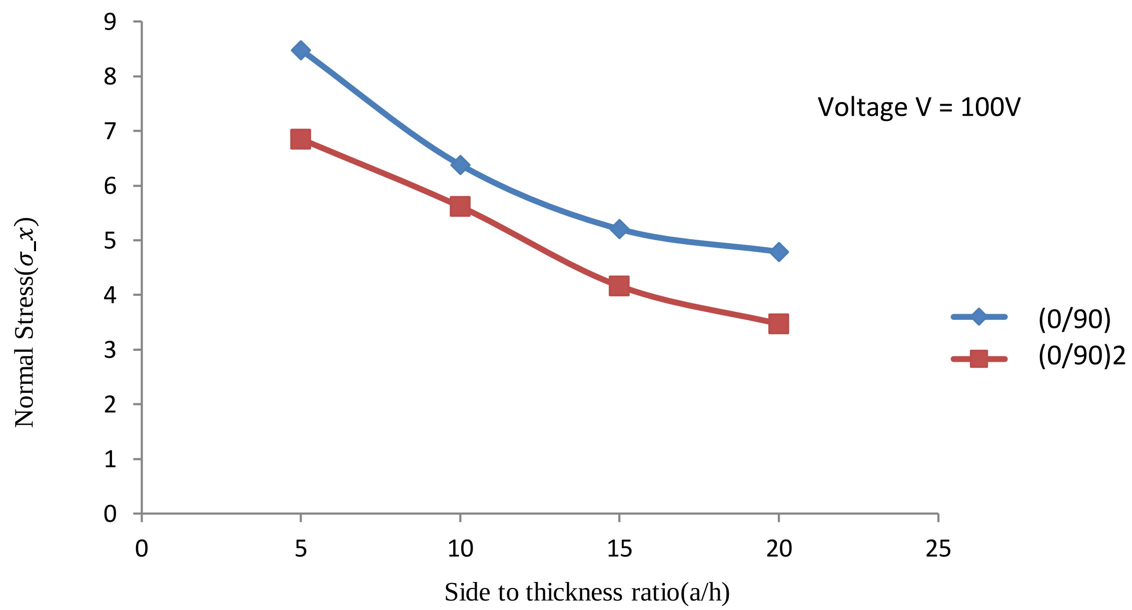

Fig.2 Non dimensionalised normal stress vs side to thickness ratio for a simply supported cross ply smart composite laminated plate.

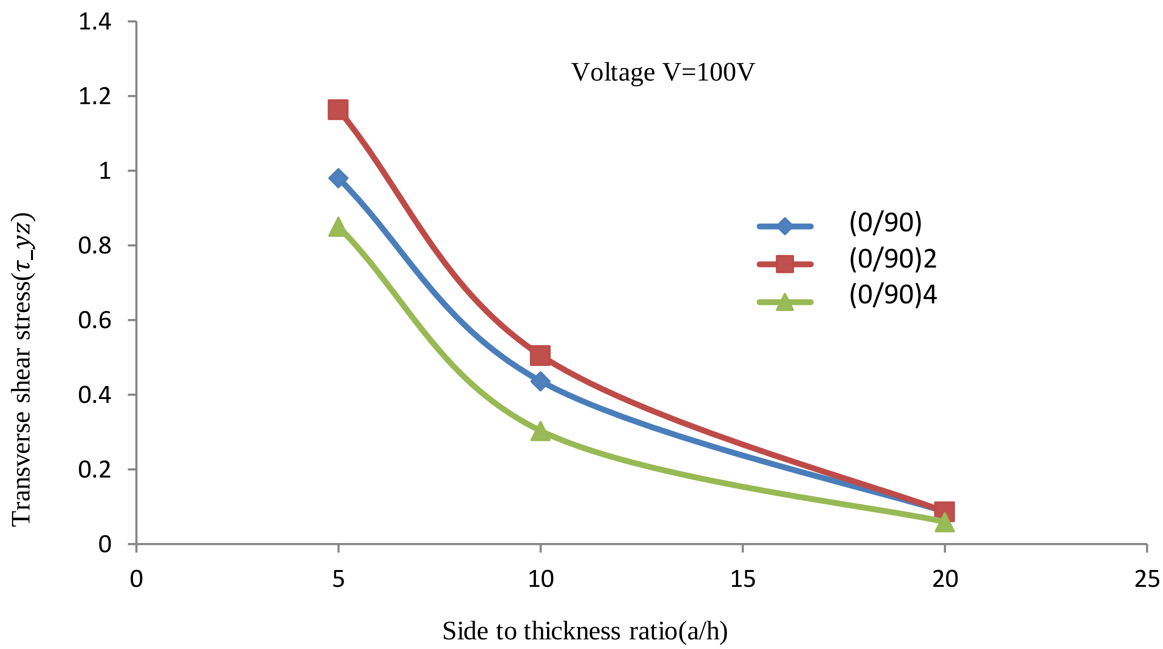

Fig:3 Non dimensionalised transverse shear stress vs side to thickness ratio for a simply supported cross ply smart composite laminated plate.

CONCLUSION

Analytical procedure is developed in this paper for thermal analysis of smart composite laminated plates subjected to electromechanical loading. The non-dimensional deflection and stresses are obtained for voltage, aspect ratios and thickness coordinates. The inclusion of the Zig Zag function in a displacement model has resulted to be more effective than the introduction of higher-order polynomials. The effect of bending stretching coupling present in 2 layered plate on stresses and a deflection is to increase the magnitude than those of 4, 6 and 10-layered plates. The higher-order theories are uniformly accurate irrespective of whether the loading is thermal or mechanical with the inclusion of Zig Zag function.

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